Name: _____

Student ID #: _____

1. What is your major?

Advanced Math / Astronomy

2. What subjects/topics have you seen previously? Check all that apply.
   - ◯ Cylindrical or Spherical coordinate systems
   - ◯ Linear Algebra
   - ◯ Gradient, divergence, curl operators
   - ◯ Partial differential equations
   - ◯ Numerical methods (integration, differentiation, interpolation, etc.)
   - ◯ Classical Mechanics
   - ◯ Electricity & Magnetism
   - ◯ Statistical Mechanics, Thermodynamics
   - ◯ Quantum Mechanics
   - ◯ Particle physics
   - ◯ Einstein's special relativity
   - ◯ Einstein's general relativity

Algebra / Geometry / Trigonometry

3. Which line is perpendicular to the line $y = 6x - 8$?
   A. $y = 6x + 1$ B. $x - 6y = 9$ C. $x + 6y = -3$ D. $y = -6x - 1$

4. Solve the following system of equations

$$\begin{aligned} 2x + 3y &= 6 \\ 4x + 9y &= 15 \end{aligned}$$

5. Solve the following equation for the semi-major axis

$$\frac{G\left(M + m\right)}{a^3} = \frac{4\pi^2}{T^2}$$

6. Solve the following equation $\phi^2 = \phi + 1$

7. When a football kicker attempts a field goal, the football usually travels around 24 m/s and has a launch angle of about 30 degrees from the horizontal.
   (a) What is the magnitude of the horizontal velocity?

   (b) What is the magnitude of the vertical velocity?

8. What is the equation for
   (a) the area of a circle

   (b) the volume of a sphere

   (c) the surface area of a sphere

   (d) the volume of a cylinder

9. The expression $\log_2 2x - 2\log_2 y + \log_2 z$ is equivalent to
   A. $\log_2\left(\dfrac{xz}{y}\right)$
   B. $\log_2\left(2x - y^2 + z\right)$
   C. $\log_2\left(\dfrac{2xz}{y^2}\right)$
   D. $\dfrac{\log_2(2x)\log_2 z}{2\log_2 y}$
   E. None of these

10. Simplify the expression $\dfrac{\left(x^4 y^{-3} z^2\right)^2}{x^3 y^5 z^{-1}}$

Calculus

11. Taylor expand $f(x) = \dfrac{1}{(1+x)^2}$

12. $\displaystyle\int_0^{\pi} \sin\theta \, \mathrm{d}\theta$

13. $\displaystyle\int \frac{x}{x+1} \, \mathrm{d}x$

14. $\dfrac{\mathrm{d}}{\mathrm{d}x} \sin\left(x^2 + x\right)$

15. $\dfrac{\mathrm{d}}{\mathrm{d}x} \dfrac{x}{\log x}$

16. $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \omega^2 y = 0$

17. $y\dfrac{\mathrm{d}y}{\mathrm{d}x} - 4x = 0$

Astronomy

18. What causes the seasons?

19. What is tidal locking?

20. What causes a solar eclipse?

21. Estimate the escape velocity of the Sun. $G \approx 7 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, $R_\odot \approx 7 \times 10^8$ m, $M_\odot \approx 2 \times 10^{30}$ kg.

22. What determines if a planetesimal becomes a terrestrial rocky planet or a gas giant?