ASTR 1040 Recitation: Compact Objects

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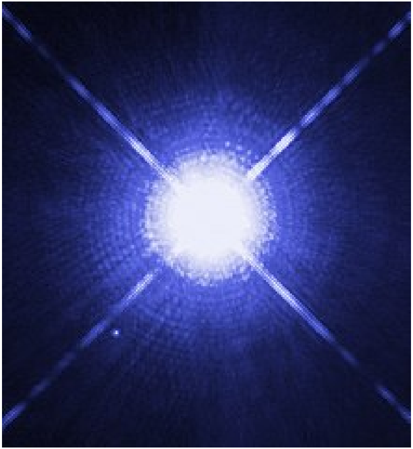
March 19 & 20, 2019

Announcements

- Last Observing: Wed, Apr 3 (8pm or 9pm at SBO)

White Dwarfs

- $R \sim R_{\oplus}$, $M \sim M_{\odot}$
- No more E generation
- Why don't they collapse?



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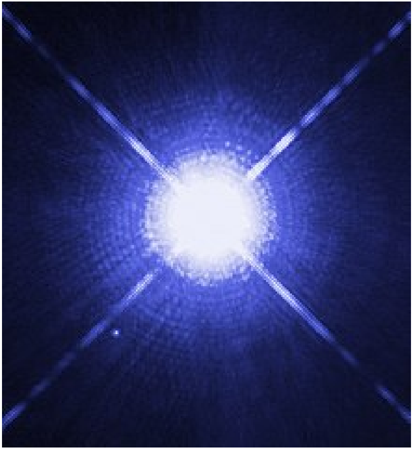
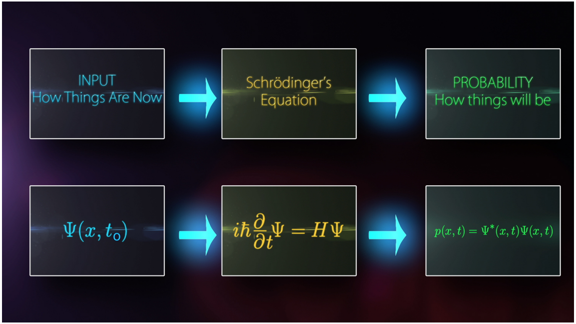


- e^- degeneracy pressure: $P_{nrel} \propto \rho^{5/3}$, $P_{rel} \propto \rho^{4/3}$

Essence of Quantum Mechanics

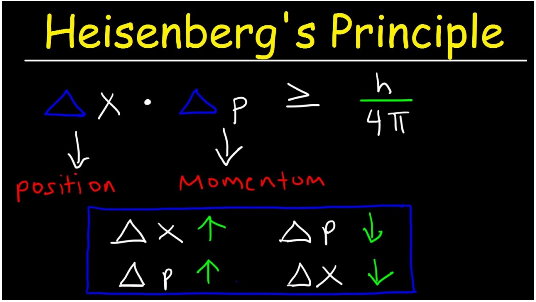
- Heisenberg Uncertainty Principle
- Pauli Exclusion Principle



- Planck's Constant: $h \approx 10^{-34}$ J s or $\hbar \equiv h/(2\pi)$ J s

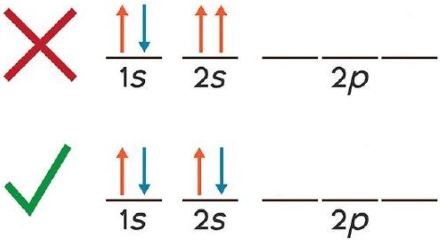
"Basic" Quantum: Heisenberg's Uncertainty

- $\Delta x \Delta p \geq \hbar/2$
- $\Delta t \Delta E \geq \hbar/2$
- p is momentum
- E is energy

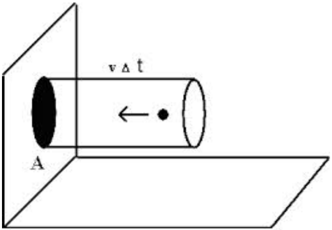


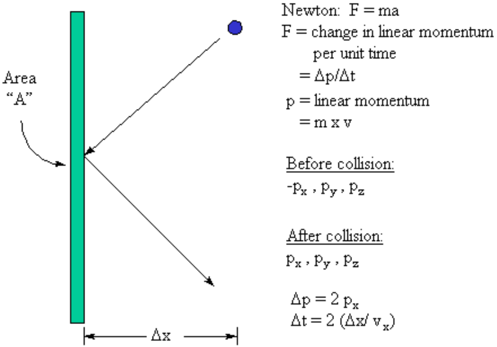
"Basic" Quantum: Pauli Exclusion

- No two fermions (protons, electrons, neutrons) can occupy the same quantum state
- Fermions have half-integer spin and Bosons (photons) have integer spin



Remember What Pressure Is?





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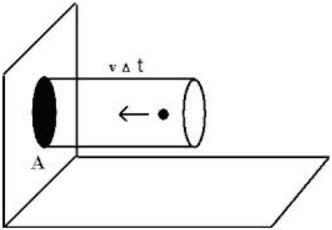

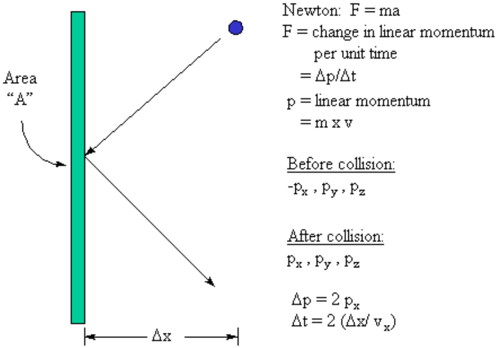

- $F = \Delta p / \Delta t = p_x v_x / \Delta x$ & $A = \Delta y \Delta z$
- $P = F/A = p_x v_x / V = p_x v_x n$

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- Electrons: $p_x = m_e v_x \qquad \Rightarrow \qquad v_x = p_x/m_e$
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- $P \approx \frac{\hbar^2}{m_e} n_e^{5/3} = \frac{\hbar^2}{m_e} \left(\frac{Z}{A} \frac{\rho}{m_H} \right)^{5/3}$ independent of T!
- $n_e = \left(\frac{\#\ e^-}{\text{nucleon}} \right) \left(\frac{\#\ \text{nucleons}}{\text{vol}} \right) = \frac{Z}{A} \frac{\rho}{m_H}$

Mass-Radius Relationship

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- What happens if we set this equal to the central P?
- $dP/dr = -\rho g$
- $dP/dr \approx P_c/R = \rho g = \frac{3M}{4\pi R^3}\frac{GM}{R^2} \quad \Rightarrow \quad P_c = \frac{3GM^2}{4\pi R^4}$
- $\frac{\hbar^2}{m_e}\left(\frac{Z}{m_H A}\frac{3M}{4\pi R^3}\right)^{5/3} = \frac{3GM^2}{4\pi R^4}$
- $MR^3 = \mathrm{Const}$, as M increases, R decreases

Electron Degeneracy Pressure, Again

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- Small R: e$^-$ confined to small space $\Rightarrow \Delta x \downarrow$, $\Delta p \uparrow$
- Velocity grows until $v \sim c$, relativity!

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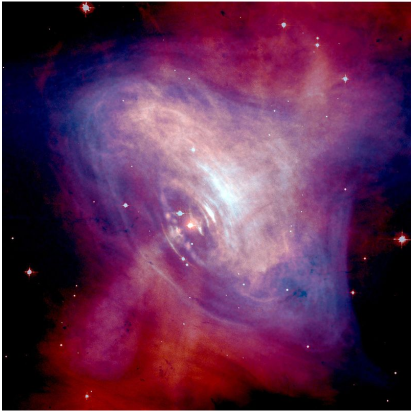
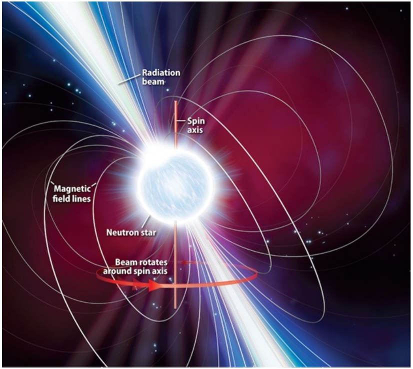
- Mass-Radius relationship: $M = \mathrm{Const} \approx 1.44 M_\odot$

Neutron Stars

- White dwarf passes Chandrasekhar limit
- Pressure loses, gravity wins
- Force nuclei and electrons to undergo "K-capture"
  - $\mathrm{p} + \mathrm{e}^- \rightarrow \mathrm{n} + \nu_e$
- Composed of almost entirely neutrons
- What holds them up against gravity?

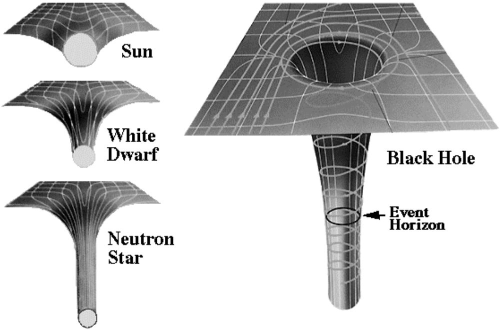
Pulsars





- Requires rotation & magnetic fields
- All pulsars are NS, not all NS are pulsars

Black Holes



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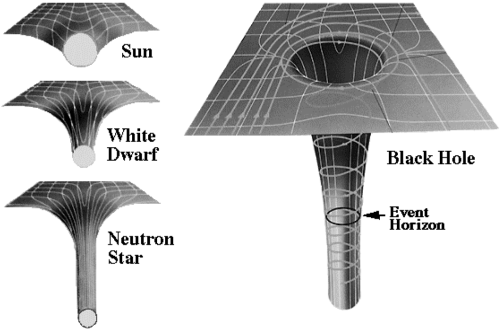

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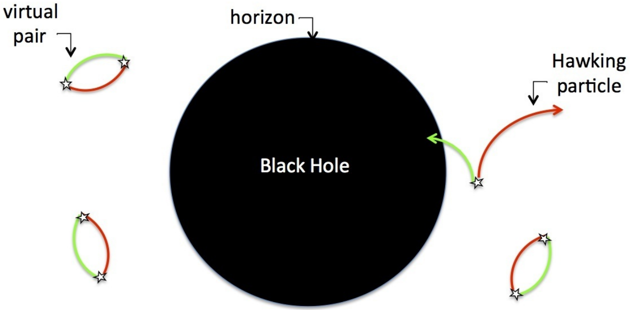
- Curved spacetime tells mass/energy how to move
- Schwarzschild radius: $r_s = \frac{2GM}{c^2}$ (actually $C/2\pi$)
- Mass, Angular momentum, & Charge

Evaporating Black Holes

- Uncertainty: $\Delta t \Delta E \geq \hbar/2$
- Produce 2 particles of total energy ΔE, but only live Δt
- What if this occurs near r_s? One escapes, one falls in



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- As seen by an observer far away, particle Bob fell in and particle Alice escaped
- Alice escaped and is real with $E > 0$
- Bob is on a bound orbit with the BH, i.e., $E < 0$
- BH swallowed a particle with $E < 0 \iff m < 0$
- Observer measures mass decrease, BH eventually evaporates
- $t_{\rm evap} \approx 2 \times 10^{67} \left(\frac{M}{M_\odot}\right)^3$ yrs

Practice Problem: Lifetime of Pulsars

Consider a rotating neutron star with $M = 2M_\odot$, radius $R = 15$ km, a period of $P = 0.1$ sec, and a rate of change in period $dP/dt \equiv \dot{P} = 3 \times 10^{-6}$ sec/yr.

$K = \frac{1}{2} I\omega^2 \qquad \omega = \frac{2\pi}{P} \qquad I = \frac{2}{5} MR^2 \qquad M_\odot = 2 \times 10^{30}$ kg

1. What is the kinetic energy of the neutron star?
2. What is the rate at which kinetic energy changes?
3. What is the lifetime of the neutron star?

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1. What is the kinetic energy? $K = 7.1 \times 10^{41}$ J
2. Rate of change in KE? $dK/dt = I\omega\dot{\omega} = -1.4 \times 10^{30}$ J/s
   $\dot{\omega} = \frac{d}{dt}\frac{2\pi}{P} = -\frac{2\pi}{P^2}\dot{P} = -\omega\frac{\dot{P}}{P}$
3. Lifetime? $\tau = \frac{K}{dK/dt} = 5.3 \times 10^{11}$ s $= 1.7 \times 10^{4}$ yrs