ASTR 1040 Recitation: Flux and Magnitudes

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Department of Astrophysical and Planetary Sciences

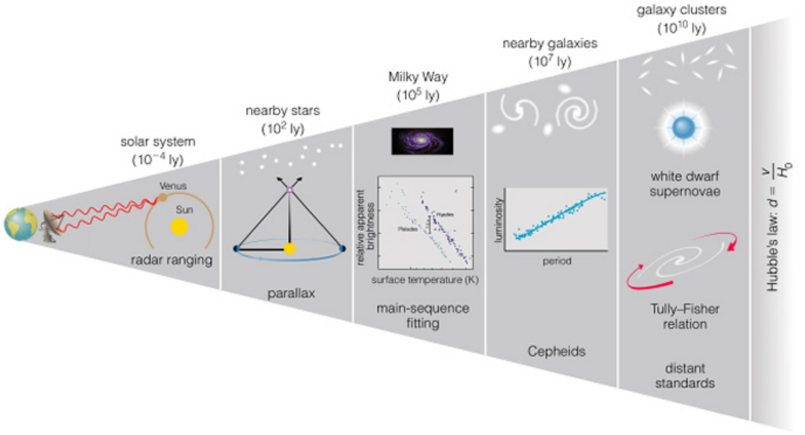
February 26 & 27, 2019

Announcements

- Next Observing: Tues, Feb 26 (8pm or 9pm at SBO)

Bow Shock

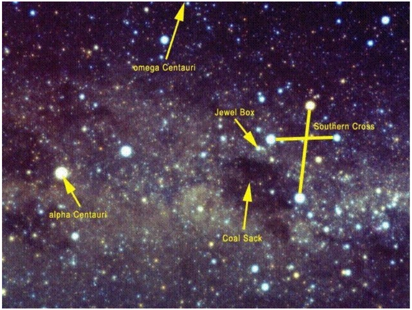
Cosmic Distance Ladder



- Distinguish between bright/distant & dim/close

Bright Stars

Apparent magnitude ≈ 1.25



β Crucis, $d \approx 280$ ly

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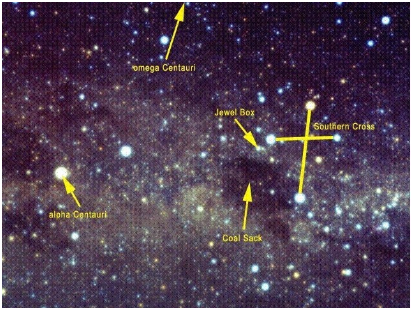

β Crucis, $d \approx 280$ ly

Denab, $d \approx 2600$ ly

Apparent Magnitudes

- Hipparchus (Ancient Greece) compiled a list of 850 stars
- Scale of 1-6, $m = 1$ is brightest, $m = 6$ is dimmest
- Difference of 5 magnitudes $\equiv$ factor of 100 in brightness

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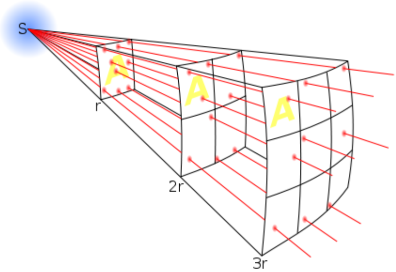
- $m_2 - m_1 = 2.5 \log_{10}(F_1/F_2)$
    - $m_\odot = -26.74$
    - $m_{\rm Moon} = -12.9$
    - $m_{\rm Jupiter} = -2.2$
    - $m_{\rm M42} = 4$
    - $m_{\rm Pluto} = 13.7$

Apparent Brightness, a.k.a., Flux

- Isotropic point source
- Each ray is radially outward
- Each ray has a unique direction
- Brightness per unit area, W/m^2



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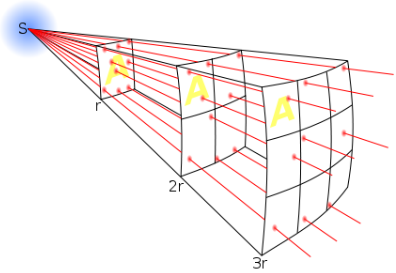

$$F = \frac{L}{4\pi r^2}$$

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- $m - M = 5 \log_{10} \left(\frac{d}{10\,\mathrm{pc}} \right)$ Distance modulus
- $m - M + 5 = 5 \log_{10} \left(\frac{d}{1\,\mathrm{pc}} \right)$ Why did I leave the 1 pc?

Blackbody Radiation

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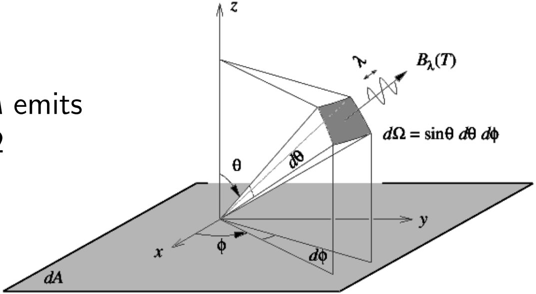
- Hotter temperatures $\Rightarrow$ more energy at all λ
- Has a local maximum, has a finite area

What is $B_\lambda(T)$, What Does It Mean?

$$B_\lambda(T) = \frac{2hc^2\lambda^{-5}}{e^{hc/\lambda kT} - 1}$$

Blackbody with area $\mathrm{d}A$ emits light into solid angle $\mathrm{d}\Omega$



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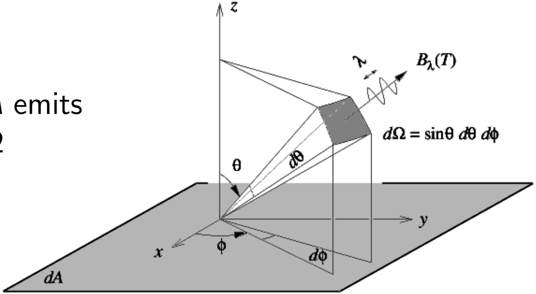
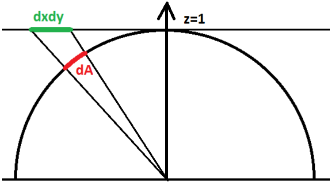


- Energy per unit time, per unit area, per unit wavelength, per unit steradian emitted by a blackbody of temperature T and surface area $\mathrm{d}A$
- J s^{-1} m^{-2} nm^{-1} str^{-1}

Quick Aside: Solid Angles



2D Angles: $\theta = s/r$

$$\mathrm{d}\theta = (\hat{r} \cdot \hat{n})\ \mathrm{d}x/r$$

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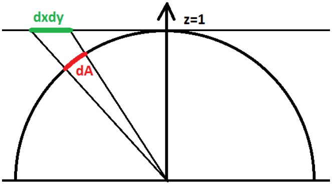

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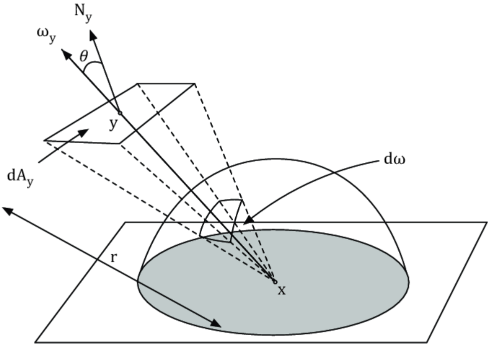

3D Angles: $\Omega = A/r^2$

$$\mathrm{d}\omega = \left(\hat{\omega}_y \cdot \hat{N}_y\right)\ \mathrm{d}A_y/r^2$$

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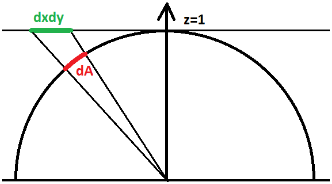

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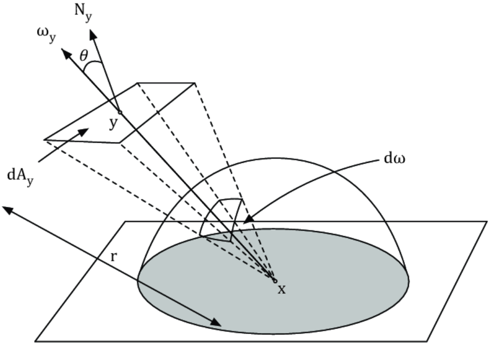

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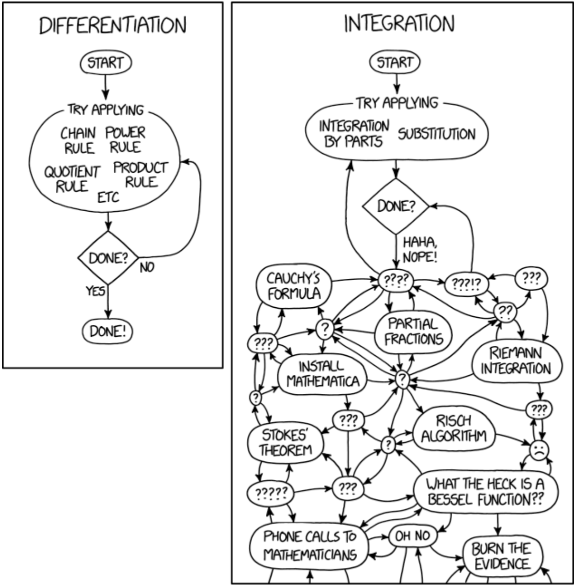
$$d\omega = \left(\hat{\omega}_y \cdot \hat{N}_y\right) dA_y/r^2$$

Standard spherical coordinates (θ, ϕ): $d\Omega = \sin\theta \, d\theta \, d\phi$

Quicker Aside: xkcd



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- Add up all the $\mathrm{d}A$ contributions over all possible solid angle directions. Then add up all wavelength contributions
- $L_\lambda = \int_\Omega \int_A B_\lambda \, \mathrm{d}A \cos\theta \, \mathrm{d}\Omega = 4\pi^2 R^2 B_\lambda = \mathrm{J\ s^{-1}\ nm^{-1}}$

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- $L = \int_\lambda L_\lambda \, \mathrm{d}\lambda = 4\pi^2 R^2 \int_\lambda B_\lambda \, \mathrm{d}\lambda =$ J s^{-1}

Effective Temperature

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- We already have L_λ and L
- $F_\lambda \,\mathrm{d}\lambda = \frac{L_\lambda}{4\pi r^2}\,\mathrm{d}\lambda = \pi B_\lambda \frac{R^2}{r^2}\,\mathrm{d}\lambda$
- $F = \int_\lambda F_\lambda \,\mathrm{d}\lambda = \pi \frac{R^2}{r^2} \int_0^\infty B_\lambda \,\mathrm{d}\lambda = \mathrm{J\ s^{-1}\ m^{-2}}$
- $\int_0^\infty B_\lambda \,\mathrm{d}\lambda = \frac{\sigma T^4}{\pi} \qquad \Rightarrow \qquad F = \frac{R^2}{r^2}\sigma T^4$

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- Suppose $r = R$, then $F = F_{\mathrm{surf}} \qquad \Rightarrow \qquad F_{\mathrm{surf}} \equiv \sigma T_{\mathrm{eff}}^4$

Practice Problem: Temperature of Ceres

Ceres is the largest object in the asteroid belt. It's gravity is strong enough to pull it into a spherical shape. Assume it acts as a perfect blackbody in order to calculate its surface T.

$d = 4.14 \times 10^{11}$ m $R = 4.73 \times 10^{5}$ m
$L_\odot = 3.84 \times 10^{26}$ W $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
$1\,\mathrm{au} = 1.50 \times 10^{11}$ m

1. What is the flux received by the asteroid?
2. What is the surface area that actually absorbs the light?
3. How much power is absorbed by the asteroid?
4. What is the Temperature of the asteroid?

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1. What is the flux? $f = \frac{L_\odot}{4\pi d^2} \approx 178.29$ W m^{-2}
2. What is the surface area? $A = \pi R^2 \approx 7.03 \times 10^{11}$ m^2
3. How much power? $P_{\mathrm{abs}} = f\,A \approx 1.25 \times 10^{14}$ W
4. What is T? $P_{\mathrm{abs}} = P_{\mathrm{emit}} = 4\pi R^2 \sigma T^4 \Rightarrow T \approx 167.4$ K