ASTR 1040 Recitation: Binary Systems

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Announcements

- Night Observing: Tues, Jan 22 (8pm or 9pm at SBO)
- Math Help Session: Wed, Jan 23 (6-8pm Duane G-126)
- Last Day to Drop without tuition/fee charges and a W grade: Jan 30
- My Office hours changed: Tu 2-4pm

Kepler's Three Laws

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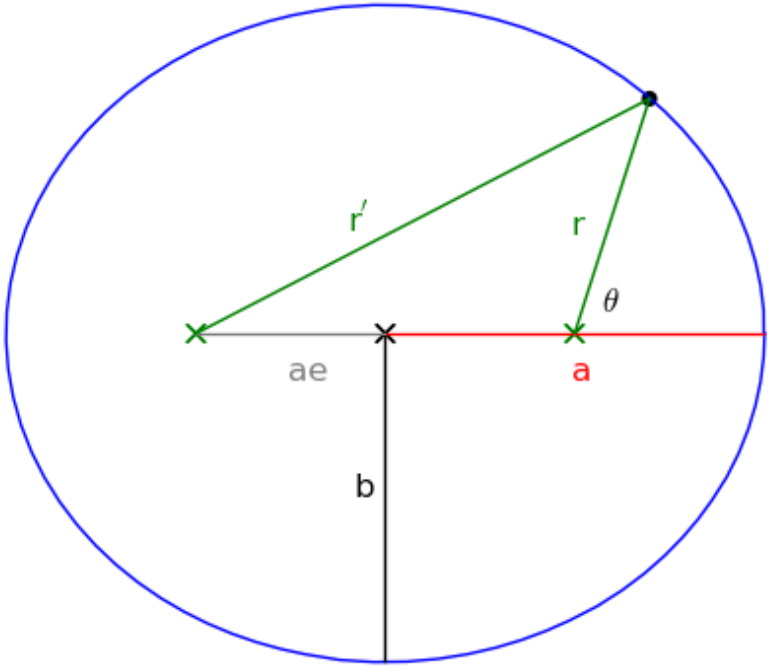
- Bound orbits are elliptical, center of mass at foci
- Equal areas in equal time
- $4\pi^2 a^3 = G\left(m_1 + m_2\right) T^2$

Visualizing the Second Law: Equal Areas

Visualizing the Third Law: $P^2 \sim a^3$

Ellipse Basics



$e \equiv$ Distance between foci / largest diameter

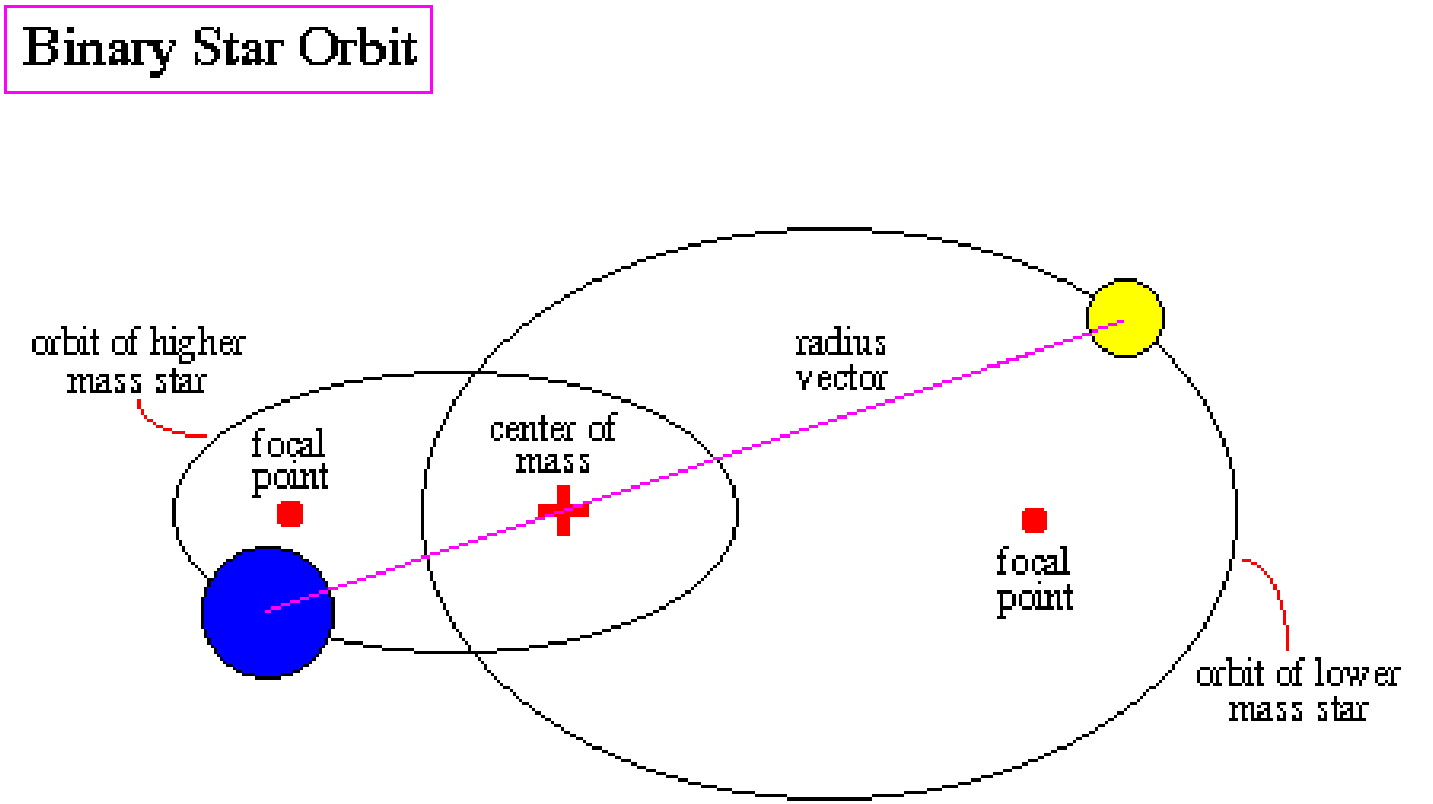
$r + r' = 2a$ $\qquad\qquad$ r_{max}, r_{min}?

Eccentricity

Binary Circular Orbits

Binary Elliptical Orbits

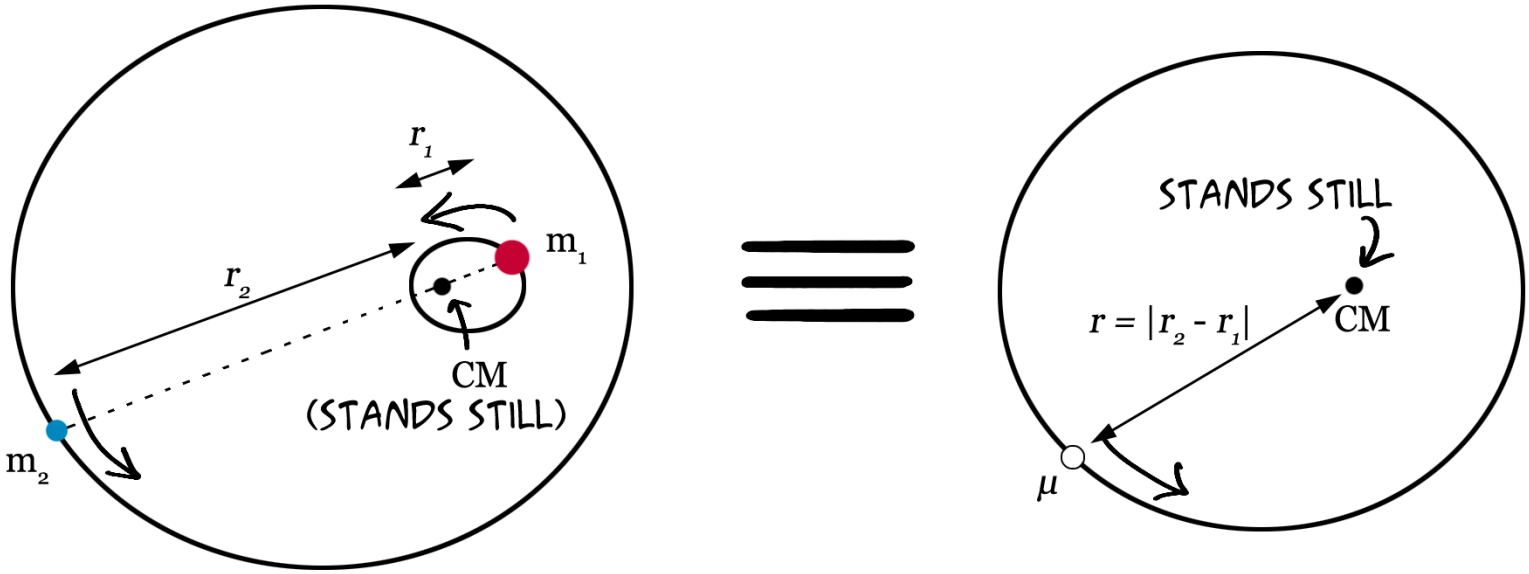
Binary Orbits



Large star has a_1, r_1 small star has a_2, r_2

Same eccentricity, $e_1 = e_2 = e$

Center of Mass & Reduced Mass



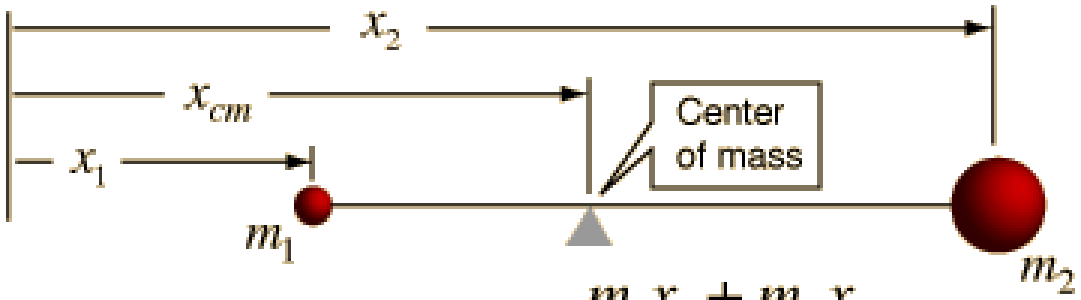
INTERACTING TWO-BODY SYSTEM

CENTER OF MASS-REDUCED MASS SYSTEM

- a_1, a_2, $r_1(t)$, $r_2(t)$
- $a_2/a_1 = r_2/r_1$

- M stationary, μ moves
- single $r(t)$, $a = a_1 + a_2$

Center of Mass & Reduced Mass



For two masses: $x_{cm} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

$$\frac{1}{\mu} \equiv \sum \frac{1}{m_i} = \frac{1}{m_1} + \frac{1}{m_2} + \dots$$

Practice Problem: Sun-Jupiter System

$M_J = 1.898 \times 10^{27}$ kg, $M_\odot = 1.989 \times 10^{30}$ kg, $T = 11.86$ yrs, $r_p = 7.405 \times 10^{11}$ m, $R_\odot = 6.957 \times 10^8$ m

- Total mass of the system, M_{tot}
- Reduced mass, μ
- Ratio of semi-major axes, $a_J/a_\odot$ (1048)
- Semi-major axis of system, a (5.199 au)
- Semi-major axes, a_J, $a_\odot$ (7.771×10^{11} m, 7.415×10^8 m)
- Compare $a_\odot$ to $R_\odot$, where is this point?
- Eccentricity, e (0.04796)